



A-Level Further Maths

Welcome!

A very warm welcome to The Crossley Heath School Sixth Form, and congratulations on choosing to study A-level Further Mathematics — a truly exciting, challenging, and rewarding subject!

You've made a fantastic decision that opens up a world of possibilities. Further Mathematics is not just about solving complex equations; it's about developing deep logical thinking, creative problem-solving, and analytical skills that are highly valued in many fields. Whether you're considering a future in engineering, computer science, physics, economics, finance, architecture, or even fields like medicine, law, and data science, Further Maths will give you a powerful edge.

Studying this subject will not only stretch your mathematical ability but also demonstrate to universities and future employers that you're ambitious, resilient, and capable of tackling advanced concepts — qualities that are essential for success in any career path.

We are genuinely excited to be part of your journey. At Crossley Heath, you'll be supported every step of the way by a passionate and experienced team who are here to help you grow in confidence, curiosity, and capability. We can't wait to explore the elegance and depth of mathematics with you and to see all that you'll achieve over the next two years.

Welcome aboard — the adventure starts here!

The Mathematics Department

Exam Board and Specification: Pearson Edexcel A-Level Further Mathematics (9FM0)

Link to Specification:

<https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/a-level-l3-further-mathematics-specification.pdf>



Section A: Vital Skills

In Further Mathematics, your algebraic skillset is as vital if not even more so than it would be for standard A level mathematicians.

Below is a selection of exercises and puzzles for you to attempt.

Task 1: Expanding brackets (30 minutes)

workspace

1. Expand and simplify
 $(2x + 3)(3x - 5)$

5. Evaluate (no calc allowed)
 $\left(2 + \frac{1}{3}\right)\left(2 - \frac{1}{3}\right)$

2. Write $(x + 3)^2 - 4$ in the form $ax^2 + bx + c$

6. Find the area of this rectangle

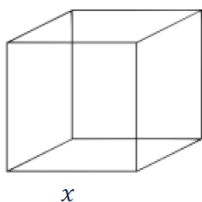
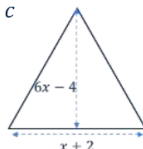


3. Expand and simplify
 $(2a + 2)(3x - 4a + 3)$

7. Expand and simplify
 $(5 - 4x)(3x + 6) + (5x - 2)(3 + 4x)$

4. Expand and simplify
 $3x(x - 3)(x + 5)$

8. Find the area of the triangle and write it in the form $ax^2 + bx + c$



Here is a cube with side lengths of x cm

The cube is going to have its lengths increased in one of three ways

Method A

Each side is increased by 2 units

Method B

One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit

Method C

One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

Can you prove which of the new solids will have the largest volume?

Task 2: Completing the square (15 minutes)

You can solve the following problem relatively easily by completing the square on each of the numerators and cancelling down. Work out this without a calculator!

$$\frac{\frac{(5^2-3^2)}{5+3} + \frac{(4^2-2^2)}{4+2} + \frac{(3^2-1^2)}{3+1}}{2}$$

workspace

Task 3: Rearranging formulae (45 minutes)

1. Make x the subject of $x - f = y + b$
2. Make y the subject $ty - x^2 = b$
3. Make c the subject $ac + d = m^2$
4. Make a the subject $x(a - e) = d$
5. Make y the subject $b(y - b) = b^2$
6. To find velocity, v , we use the formula $v^2 = u^2 - 2as$
Rearrange to find s
7. The area of a sector of a circle is given by $A = \frac{\theta\pi r^2}{360}$ Express θ in terms of A, π and r
8. Make x the subject $m(y - x) = t$

workspace



Task 4: Straight line graphs (30 minutes)

Can you sort the cards into pairs under the following headings:

- These lines are perpendicular
- These lines have the same x intercept
- These lines have the same y intercept
- These lines are parallel
- These lines go through the point (1,5)
- These lines...

$$3y = 2x - 8$$

$$y = -(x + 8)$$

$$y = 4x + 4$$

$$2y + x = 4$$

$$y = 6x - 4$$

$$y = 8x - 3$$

$$y + x + 8 = 0$$

$$2y = 8x + 3$$

$$4y = x + 3$$

$$2y + 8 = 3x$$

$$y + 6x = 11$$

$$y + 4x + 6 = 0$$

workspace

Task 5: Inverse functions (30 minutes)

Original function

$$f(x) = 3x + 2$$

Inverse function

$$f^{-1}(x) = \frac{x-2}{3}$$

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$

5. $f(x) = \frac{2}{3}x + 3$

2. $f(x) = 4x + 7$

6. $f(x) = 3 - 2x$

3. $f(x) = \frac{x}{2} + 1$

7. $f(x) = x^2$

4. $f(x) = \frac{x+2}{3}$

8. $f(x) = \sqrt{x+1}$

workspace



Task 6: Equivalent quadratics (30 minutes)

Sort the expressions below in to 4 sets of 4 equivalent expressions.

$x^2 - 25$	$2x^2 - 2$
$(x + 5)(x + 6) - x - 55$	$(x + 5)(x - 5)$
$2(x^2 - 1)$	$(x + 5)^2 - 10x - 50$
$2(x + 3)(x - 1)$	$2(x + 1)(x - 1)$
$(x + 5)^2 - 50$	$2(x + 2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x + 5)(x - 5) + 10x$
$2(x + 1)^2 - 8$	$(x - 5)(x + 6) - x + 5$
$x^2 + 10x - 25$	$2(x + 1)^2 - 4(x - 1)$

workspace

Task 7: Mean squares (30 minutes)

workspace

- Take two positive values greater than 1
 - Find the mean of the two values
 - Square it
- Then
- Take the same two values
 - Square them
 - Find the mean of the squares

Which value is greater?
Is this always true?
Can you prove it?



Task 8: Forming the formulae (15 minutes)

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right) = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

workspace

Using your rearranging skills can you prove each of the following

If $a = \frac{b}{b+c}$

Show that $\frac{a}{1-a} = \frac{b}{c}$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

Given that $\tan x = \frac{\sin x}{\cos x}$ and $\tan^2 x = \tan x \times \tan x$

If $\sin x = \frac{1}{2}(2 + \sqrt{3})$ and $\cos x = \frac{1}{2}(2 - \sqrt{3})$

Find an expression for $\tan x$ in the form $a + b\sqrt{3}$

Task 9: Prove it! (45 minutes)

workspace



Section B: Research Task (4 hours)

Some rules that you have heard about at GCSE are not quite 100% of the picture.

One example is that you cannot square root a negative number. This is true in the number system you are currently aware of, but it is possible if you broaden the range of numbers you are allowed to work with. These numbers are called complex numbers and you can start to learn about these at the following links.

<https://nrich.maths.org/13167>

<https://nrich.maths.org/13129>

<https://nrich.maths.org/13131>

This is a good starting point but if you want to learn more, then explore the rest of the website and do some research in other places such as YouTube!

Speaking of YouTube, if you want to start learning about more abstract interesting mathematics, start watching videos by Numberphile. This channel has been going for years and trying to follow their videos with massively develop your skills. They are accessible but definitely challenging!

Channel Link - <https://www.youtube.com/user/numberphile>

Suggested videos

Golden Ratio - https://www.youtube.com/watch?time_continue=480&v=sj8Sg8qnjOg

The scientific way to cut a cake - <https://www.youtube.com/watch?v=wBU9N35ZHIw>

The sum of 1,2,3,4,... - <https://www.youtube.com/watch?v=w-l6XTVZXww>



YouTube