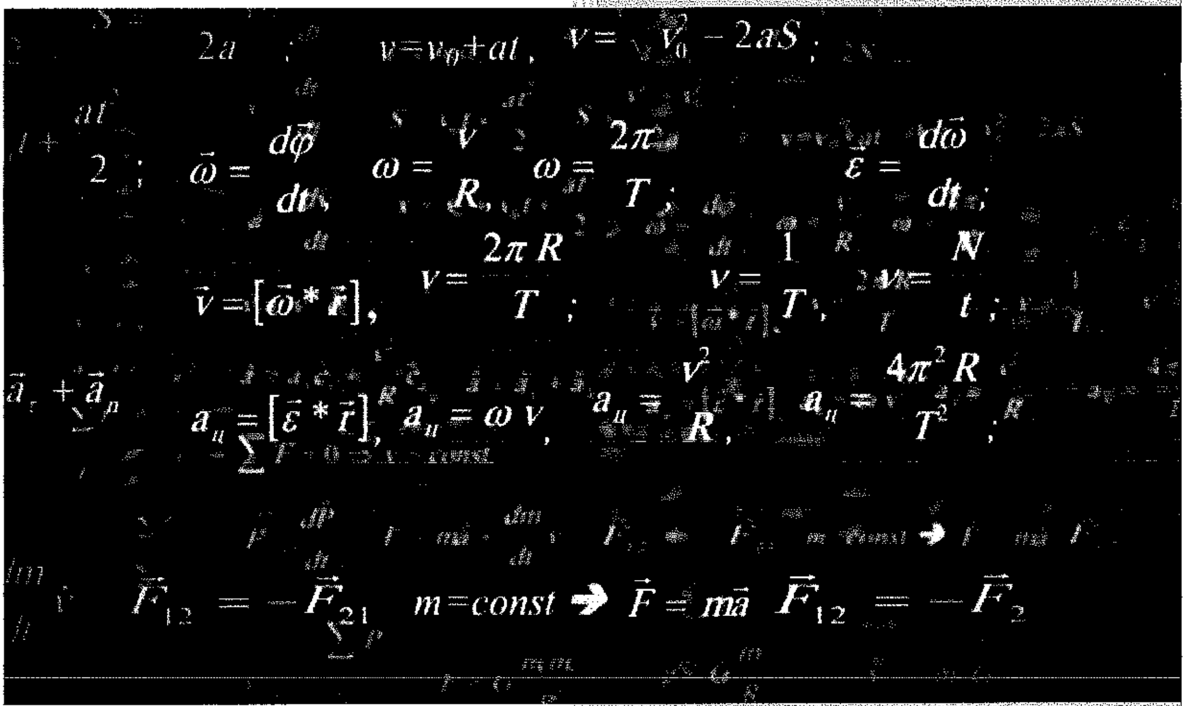
A Level Physics Bridging Work

Booklet 1: Mathematics



Name:

Section 1: Mathematics

Instructions:

As you work through this section complete the Student tasks on each page. Write your answers on lined paper – you will need them for your first Physics lesson in September.

Symbols and SI Units

One of the biggest jumps between GCSE Physics and AS Physics is in the way things are written down. At AS level, you're expected to start using standard scientific notation...

Standard notation means:

- using the conventional symbols for quantities, e.g. a temperature should always have the symbol T
- writing all quantities in terms of SI units (Système International)
- writing very large and very small numbers in standard form (e.g. 10^6 or 10^{-3})

That last point means you shouldn't have to worry about meeting nasty units like microseconds (μs) and megajoules (MJ) in exams. They'll usually be written as 10^{-6} s and 10^6 J instead. (If you can't remember how standard index form works, look it up in one of our maths guides... or phone a friend.)

You still need to learn the unit prefixes though, because textbooks use them — see below.

The table below lists the different quantities you'll meet in this book, with their standard symbols and units.

Quantity	Symbol	Unit	
		Name	Symbol
Displacement (distance)	s	metre	m
Time	t	second	s
Velocity (speed)	v	metre per second	ms^{-1}
Acceleration	a	metre per second squared	ms^{-2}
Mass	m	kilogram	kg
Force	F	newton	N
Gravitational field strength	g	newton per kilogram	Nkg^{-1}
Energy	E	joule	J
Power	P	watt	W
Frequency	f	hertz	Hz
Wavelength	λ	metre	m
Temperature	T	kelvin	K
Charge	Q	coulomb	C
Electric current	I	amp	A
Potential difference	V	volt	V
Resistance	R	ohm	Ω

At A Level, units like m/s are written ms^{-1}. This is just index notation. (If it doesn't make sense to you, look up 'rules of indices' in a maths book.)

If you write the unit out in full, it should be all lower case — always.

If the unit comes from someone's name, the symbol should start with a capital letter e.g. Hz.

Here are the unit prefixes for those long numbers:

Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Of course, what they don't tell you at AS is that people in the real world don't use standard scientific notation. A lot of textbooks that you'll be using will stick to the old prefix system for units.

And you'll also meet odd units like parsecs, electonvolts and atomic mass units, which are definitely not SI.

1 Manipulating Equations

1.1 INTRODUCTION

Physicists are often concerned with the relationships between quantities and the way in which one quantity changes as another is altered. Relationships could be written out in words, for example:

force is equal to mass multiplied by acceleration

but for conciseness these relationships are usually written as an algebraic equation, with one letter to represent each quantity, for example

$$F = ma$$

Even quite simple relationships would be quite difficult to follow if they were not written in this way, for example, compare

distance travelled is equal to initial velocity multiplied by time taken plus one half of the acceleration multiplied by the square of the time taken

with

$$s = ut + \tfrac{1}{2}at^2$$

Apart from conciseness, equations have the advantage that they can easily be manipulated using the rules of algebra, which is often necessary to help solve problems.

1.2 SIGNS AND SYMBOLS

In order to understand the language of algebra and arithmetic, you need to know the symbols listed in Table 1.1. The meaning of some of these symbols will made clearer in later chapters, but all the necessary signs and symbols are included here for convenience.

Table 1.1 Algebraic symbols

Symbol	Meaning	Example	
$<$	less than	$x < 20$	x is less than 20
$>$	greater than	$y > 15$	y is greater than 15
$\ll$	much less than	$z \ll 1$	z is much less than 1
$\gg$	much greater than	$w \gg 5$	w is much greater than 5
$\leqslant$	less than or equal to	$x \leqslant 1$	x is less than or equal to 1
$\geqslant$	greater than or equal to	$y \geqslant 1$	y is greater than or equal to 1
$=$	equal to	$w = 5$	w has a value of 5
$\approx$	approximately equal to	$\pi^2 \approx 10$	π^2 is approximately equal to 10
$\neq$	not equal to	$z \neq a$	z does not equal a
$\propto$	proportional to	$F \propto a$	F is proportional to a
$\bar{x}$	mean or average	$\bar{x} = 6$	the mean value of x is 6

1.3 THE BASIC RULES OF ALGEBRA AND ARITHMETIC

In order to be able to manipulate algebraic equations as well as carrying out numerical calculations, you must know and understand the basic rules of algebra. Four useful examples are shown in Table 1.2.

Table 1.2 Simple algebraic rules

	Example
$by = y + y + \ldots$(b times)	$3y = y + y + y$
$ay + by = (a + b)y$	$2y + 3y = 5y$
$\frac{1}{\left(\frac{a}{y}\right)} = \frac{y}{a}$	$\frac{1}{\left(\frac{2}{y}\right)} = \frac{y}{2}$
$\frac{\left(\frac{b}{z}\right)}{\left(\frac{a}{y}\right)} = \frac{by}{az}$	$\frac{\left(\frac{3}{z}\right)}{\left(\frac{2}{y}\right)} = \frac{3y}{2z}$

1.4 HANDLING INDICES

y^a means y multiplied by itself a times. For example, 4^3 means $4 \times 4 \times 4 = 64$.

a is called the index, or the power to which y is raised. (We say, for example 'y to the power of a' or '4 to the power of 3'.)

Many scientific equations contain indices. The rules for manipulating indices are shown in Table 1.3.

Table 1.3 Handling indices

	Algebraic example	Numerical example
$y^{-n} = \frac{1}{y^n}$	$y^{-3} = \frac{1}{y^3}$	$4^{-3} = \frac{1}{4^3} = \frac{1}{64} = 0.0156$
$y^{\frac{1}{a}} = \sqrt[a]{y}$	$y^{\frac{1}{2}} = \sqrt{y}$	$4^{\frac{1}{2}} = \sqrt{4} = 2$
$y^a + y^b$	cannot be simplified	$4^2 + 4^3 = 16 + 64 = 80$
$y^a \times y^b = y^{a+b}$	$y^2 \times y^3 = y^5$	$4^2 \times 4^3 = 4^5 = 1024$
$\frac{y^a}{y^b} = y^{a-b}$	$\frac{y^2}{y^3} = y^{2-3} = y^{-1}$	$\frac{4^2}{4^3} = 4^{-1} = \frac{1}{4} = 0.25$
$(y^a)^b = y^{a \times b}$	$(y^2)^3 = y^6$	$(4^2)^3 = 4^6 = 4096$
$y^{\frac{b}{a}} = \sqrt[a]{y^b}$	$y^{\frac{3}{2}} = \sqrt[2]{y^3}$	$4^{\frac{3}{2}} = \sqrt[2]{4^3} = 8$
$y^0 = 1$		$4^0 = 1$

✍ *Student task 1.1*

a) Simplify the following:

i) $y^6 \times y^7$ ii) $y^8 \div y^5$

iii) $(y^3)^4$ iv) $3y^2 \times 4y^3$

b) Calculate the values of the following:

i) 3^4 ii) $8^{\frac{1}{3}}$ iii) 5^{-2}

iv) $9^{\frac{3}{2}}$ v) $16^{-\frac{1}{2}}$ vi) 6^0

1.5 CHANGING THE SUBJECT OF AN EQUATION

In the equation $v = u + at$, v is known as the subject of the equation. You can calculate a value for v if you know the values of u, a and t by substituting these values into the equation. However, you may need to calculate the value of t, for example, so you would need to make t the subject of the equation.

Changing the subject of an equation is based on the general principle:

> Whatever change you make to one side of the equation, you must make the same change to the other side.

The following examples should make this clear.

i) Addition and subtraction

The equation for resistors in series is:

$$R_T = R_1 + R_2$$

To make R_1 the subject of the equation, we subtract R_2 from the right hand-side, leaving only R_1, so we also subtract R_2 from the left-hand side, giving:

$$R_T - R_2 = R_1$$

or

$$R_1 = R_T - R_2$$

Effectively, we have moved R_2 to the other side of the equation and changed its sign from positive to negative.

> If something which is added to one side of an equation is moved to the other side, it changes its sign from positive to negative. Conversely, if something which is subtracted from one side of an equation is moved to the other side, it changes its sign from negative to positive.

> ✍ *Student task 1.2*
>
> a) Make u the subject of $v = u + at$
>
> b) Make E_k the subject of $hf = \varphi + E_k$
>
> c) Make $\frac{1}{v}$ the subject of $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

ii) Multiplication and division

This is by far the most common kind of equation in Physics.

The equation for power in an electric circuit is:

$$P = VI$$

To make V the subject of the equation, we divide the right hand side by I, so we must also divide the left hand side by I giving:

$$\frac{P}{I} = V$$

Effectively we have moved I to the other side of the equation and changed its sign from 'multiply' to 'divide'.

> If something which multiplies one side of an equation is moved to the other side, it divides that side of the equation (it moves from being a *numerator* to being a *denominator*). Conversely, if something which divides one side of an equation moves to the other side, it multiplies that side of the equation.

> ✍ *Student task 1.3*
>
> a) Make B the subject of $F = BIl$
>
> b) Make ρ the subject of $R = \frac{\rho l}{A}$
>
> c) Make V the subject of $C = \frac{Q}{V}$

iii) Squares and square roots

If $x^2 = a + b$

then to find x we must find the square root of both sides, so

$$x = \sqrt{a + b}$$

If $\sqrt{y} = c + d$

then to find y we must square both sides, so

$$y = (c + d)^2$$

iv) Further examples

Example a)

The efficiency of a heat engine is given by the equation:

$$E = \frac{T_1 - T_2}{T_1}$$

Make T_2 the subject of the equation.

Since the whole of the right-hand side of the equation is divided by T_1, the first step is to take this T_1 to the left-hand side:

$$ET_1 = T_1 - T_2$$

Now $-T_2$ can be taken to the left, and ET_1 to the right:

$$T_2 = T_1 - ET_1$$

Finally, T_1 could be taken out as a common factor on the right:

$$T_2 = T_1\,(1 - E)$$

If we wanted to make T_1 the subject of the equation, we would have to go through the same steps, and finally take the $(1 - E)$ to the left-hand side, leaving just T_1 on the right:

$$\frac{T_2}{(1 - E)} = T_1$$

> **❋ *Pitfall 1.1***
>
> In this example, it is easy to forget that T_2 is divided by T_1, resulting in the following:
>
> $$E = \frac{T_1 - T_2}{T_1} \quad \Rightarrow \quad E + T_2 = \frac{T_1}{T_1} = 1$$
>
> which is clearly nonsense!

Example b)

The frequency of oscillation of a mass on a spring is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Make k the subject of the equation.

Since m is inside the square root sign, we start by moving 2π to the left-hand side:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad 2\pi f = \sqrt{\frac{k}{m}}$$

Now both sides of the equation can be squared to remove the square root:

$$2\pi f = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad (2\pi f)^2 = \frac{k}{m}$$

Finally, m can be moved to the opposite side of the equation.

$$(2\pi f)^2 = \frac{k}{m} \quad \Rightarrow \quad m(2\pi f)^2 = k$$

> **❋ *Pitfall 1.2***
>
> Beware of accidentally removing a variable from within a square root. A possible mistake in the previous example is:
>
> $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad mf = \frac{1}{2\pi}\sqrt{k}$$
>
> However, the following is correct, although not particularly easy to work with:
>
> $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad \sqrt{m}\,f = \frac{1}{2\pi}\sqrt{k}$$
>
> However, it is possible to confuse the following:
>
> $$\sqrt{m}\,f \quad \text{with} \quad \sqrt{mf}$$
>
> especially with hasty handwriting. It is much safer to write this as:
>
> $$f\sqrt{m}$$

Example c)

Make u the subject of the following equation of motion:

$$v^2 = u^2 + 2as$$

The necessary stages are:

$$v^2 = u^2 + 2as \quad \Rightarrow \quad u^2 = v^2 - 2as$$

$$u^2 = v^2 - 2as \quad \Rightarrow \quad u = \sqrt{v^2 - 2as}$$

> **❋ *Pitfall 1.3***
>
> $\sqrt{v^2 - 2as}$ does not equal $v - \sqrt{2as}$

Example d)

It is often necessary to move several variables to achieve what is required, but the principles remain the same.

The magnetic field in a long solenoid is given by:

$$B = \frac{\mu_0 NI}{l}$$

To make I the subject of the equation, we move l to the left, where it becomes part of the numerator ('from the bottom to the top of the fraction'), while $\mu_0 N$ moves to the left to become part of the denominator ('from the top to the bottom of the fraction'), giving:

$$\frac{Bl}{\mu_0 N} = I \quad \text{or} \quad I = \frac{Bl}{\mu_0 N}$$

✍ *Student task 1.4*

a) Make l the subject of $R = \frac{\rho l}{A}$

b) Make x the subject of $\frac{\lambda}{x} = \frac{s}{l}$

c) Make v the subject of $F = \frac{mv^2}{r}$

d) Make r the subject of $F = \frac{Gm_1m_2}{r^2}$

e) Make E the subject of $c = \sqrt{\frac{E}{\rho}}$

f) Make T the subject of $f = 2\pi\sqrt{\frac{T}{\mu}}$

g) Make L the subject of $f_r = \frac{1}{2\pi\sqrt{LC}}$

1.6 SIMULTANEOUS EQUATIONS

If you have two equations relating two different unknown quantities, these equations are known as 'simultaneous'. They have a limited use in A-level Physics.

A simple example of a pair of simultaneous equations is:

$$x + 7y = 38$$
$$x + 3y = 18$$

To solve these equations (that is, find the values of x and y), we need to eliminate one of the variables; in this case it is easiest to eliminate x by subtracting the second equation from the first:

$$x + 7y = 38$$
$$x + 3y = 18$$
$$(x - x) + (7y - 3y) = (38 - 18)$$
$$4y = 20$$
$$\therefore y = 5$$

This value for y can now be substituted back into the first of the pair of equations to find a value for x:

$$x + (7 \times 5) = 38$$
$$x + 35 = 38$$
$$\therefore x = 3$$

Example

Two cars are moving along a road; car 1 is moving at a steady speed of 20 m/s; car 2 is 150 m in front of car 1 and moving at a steady speed of 15 m/s.

How much time passes before car 1 catches up with car 2, and how far will it have travelled?

Let s be the displacement when they meet, measured from the starting position of the first car, v_1 and v_2 the respective speeds of the cars and t the time at which they meet.

For car 1: $s = v_1 t$ $\quad \therefore s = 20t$

For car 2: $s = v_2 t + 150$ $\quad \therefore s = 15t + 150$

Subtracting the two equations gives:

$$0 = 5t - 150$$
$$5t = 150$$
$$\therefore t = 30$$

Car 1 passes car 2 after 30 seconds.

By substituting this value into the first equation, we can see that car 1 travels

$$20 \text{ m/s} \times 30 \text{ s} = 600 \text{ m}$$

before overtaking car 2.

2.3 STANDARD FORM

Many of the numbers we meet in Physics are either very large or very small.

For example, the mass of the earth is:

5 980 000 000 000 000 000 000 000 kg

while the charge on an electron is:

0.000 000 000 000 000 000 16 C

It is clearly ludicrous to try to do calculations with numbers written like this. You cannot key them into your calculator for a start, and it would be very easy to make a mistake with the number of zeros. In order to overcome this problem, we write numbers in standard form. A number is expressed as a number between 1 and 10 multiplied by an appropriate power of 10.

For example, 347 can be written as 3.47×100, or 3.47×10^2.

3.47×10^2 is known as **standard form.**

Using standard form, the mass of the earth can be written as 5.98×10^{24} kg, and the charge on an electron as 1.6×10^{-19} C.

This is clearly a much more convenient way to write such numbers, but there are other good reasons for using standard form, outlined in the following sections. It is very important that you should be able to handle standard form easily.

2.4 CONVERTING TO STANDARD FORM

To convert 37 800 to standard form.

The number between 1 and 10 which is needed is 3.78.

37 800 is $3.78 \times 10\,000$. 10 000 is 10^4 (see Table 2.1) so 37 800 in standard form is 3.78×10^4.

An alternative approach is to say that to go from 3.78 to 37 800, the numbers must move 4 places to the left, as follows:

3.78
37.8
378.0
3 780.0
37 800.0

Therefore the power of 10 required is 4, and 37 800 in standard form is 3.78×10^4.

To convert 0.0052 to standard form.

The number between 1 and 10 which is needed is 5.2. 0.0052 is 5.2×0.001. 0.001 is 10^{-3}, so 0.0052 in standard form is 5.2×10^{-3}.

Alternatively, to go from 5.2 to 0.0052 the numbers must be moved 3 places to the right, therefore the required power is -3, and 0.0052 in standard form is 5.2×10^{-3}.

Table 2.1

10^6	=	1 000 000
10^4	=	10 000
10^3	=	1000
10^1	=	10
10^0	=	1
10^{-1}	=	0.1
10^{-3}	=	0.001
10^{-6}	=	0.000 001

✍ ***Student task 2.4***

Convert the following numbers to standard form:

a) 3470
b) 68 000 000
c) 27
d) 0.594
e) 0.000 92
f) 264.2

2.8 UNITS AND STANDARD FORM

Quantities are often quoted in standard multiples or divisions of the basic units. The standard prefixes, together with the factor by which they multiply the basic unit, are listed in table 2.2 below.

Table 2.2

Name	Abbreviation	Multiplying factor
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

For example

1 millimetre = 1 thousandth of a metre

= 1×10^{-3} m

4.6 kW = 4.6×10^{3} W

Student task 2.7

Write the following in standard form.

a) 7.21 μV
b) 42 GW
c) 0.39 nF
d) 592 MJ
e) 0.019 pA

Quantities often have to be written in terms of the basic unit before a calculation can be performed. This can easily be done by inserting the appropriate power of 10 in place of the prefix, and is best done by writing out a table of the relevant information before you start, as in the example in the next column. (It is a good habit to write out a table of information for anything but the simplest calculation whether or not you have to do any unit conversions.)

Example

An electric power cable has a diameter of 6.0 mm and is made of a material of resistivity 27 nΩ m. What is the resistance of a 1.0 km length of the cable?

Table of information:

Diameter = 6.0 mm	= 6.0×10^{-3} m
Hence radius	= 3.0×10^{-3} m
Length = 1.0 km	= 1.0×10^{3} m
Resistivity = 27 nΩ m	= 27×10^{-9} Ω m
Resistivity	= 2.7×10^{-8} Ω m

Calculation:

$$\text{Cross section area (CSA)} = \pi r^2$$

$$CSA = \pi \times 3.0 \times 10^{-3} \times 3.0 \times 10^{-3}\ \text{m}^2$$

$$CSA = 2.827 \times 10^{-5}\ \text{m}^2$$

$$\text{Using } R = \frac{\rho l}{A}$$

$$\left(\text{Resistance} = \frac{\text{resistivity} \times \text{length}}{\text{area}}\right)$$

$$R = \frac{2.7 \times 10^{-8}\ \Omega\,\text{m} \times 1.0 \times 10^{3}\ \text{m}}{2.827 \times 10^{-5}\ \text{m}^2}$$

$$R = 0.96\ \Omega$$

The resistance is 0.96 Ω.

Pitfall 2.7

Mistakes are often made in converting areas and volumes into the basic units, such as square millimetres into square metres.

Note

10^4 square centimetres = 1 square metre.
10^6 square millimetres = 1 square metre
10^6 cubic centimetres = 1 cubic metre
10^9 cubic millimetres = 1 cubic metre

Student task 2.8

Calculate the resistance between the faces of a wafer of pure silicon of thickness 5 mm and cross section area 8.0 cm². The resistivity of silicon is 60 Ω m.

(Unlike the question above, you have been told the area, not the diameter.)